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Holographic description of quantum black hole on a computer

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Abstract

The discovery of the fact that black holes radiate particles and eventually evaporate led Hawking to pose the well-known information loss paradox. This paradox caused a long and serious debate since it claims that the fundamental laws of quantum mechanics may be violated. A possible cure appeared recently from superstring theory, a consistent theory of quantum gravity: if the holographic description of a quantum black hole based on the gauge/gravity duality is correct, the information is not lost and quantum mechanics remains valid. Here we test this gauge/gravity duality on a computer at the level of quantum gravity for the first time. The black hole mass obtained by Monte Carlo simulation of the dual gauge theory reproduces precisely the quantum gravity effects in an evaporating black hole. This result opens up totally new perspectives towards quantum gravity since one can simulate quantum black holes through dual gauge theories.

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Introduction

In 1974 Hawking realized that a black hole should radiate particles as a perfect blackbody due to quantum effects in the surrounding space, and that the black hole should eventually evaporate completely [1, 2]. This discovery made more accurate the close analogy between the laws of black hole physics and those of thermodynamics, which was pointed out originally by Bekenstein [3]. However, it also caused a long scientific debate (see, for instance, refs [4] and [5]) concerning *the information loss paradox* [6, 7], which can be described roughly as follows. Suppose one throws a book into a black hole. While the black hole evaporates, all we observe is the blackbody radiation. Therefore, the information contained in the book is lost forever. This statement sharply conflicts with a basic consequence of the law of quantum mechanics that the information of the initial state should never disappear. Then the question is whether the law of quantum mechanics is violated or Hawking's argument should somehow be modified if full quantum effects of gravity are taken into account.

In order to resolve this paradox, it is necessary to construct microscopic states of the black hole and to give a statistical-mechanical explanation for the black hole entropy. This seems quite difficult within general relativity because of the no-hair theorem, which states that black holes are characterized by only a few parameters. In the mid 1990s, however, superstring theory succeeded in explaining the entropy of "extremal black holes", a special class of black holes, which do not evaporate [8]. Superstring theory contains not only strings but also solitons called D-branes [9] as fundamental objects. Bound states of D-branes can be so heavy that they look like "black objects" from a distant observer. In fact there are many bound states, which look like the same black hole. These bound states can be interpreted as the microscopic states of the black hole, and the number of such states has been shown to explain precisely the black hole entropy.

However, the paradox still remains since a complete description of an evaporating black hole has not yet been established. A key to really resolve the paradox is provided by Maldacena's *gauge/gravity duality conjecture* [10] (Fig. 1), which may be viewed as a concrete realization of *the holographic principle* proposed by 't Hooft [11] and Susskind [12]. This conjecture relates various black holes made of D-branes in superstring theory to strongly coupled gauge theories, in which the absence of information loss is manifest. In this article we provide the first quantitative evidence for the gauge/gravity duality at the level of quantum gravity. We perform Monte Carlo simulation of the dual gauge theory in the parameter regime that corresponds to a quantum black hole. Our results agree precisely with a prediction for an evaporating black hole including quantum gravity corrections. Thus we find that the dual gauge theory indeed provides a complete description of the quantum nature of the evaporating black hole.

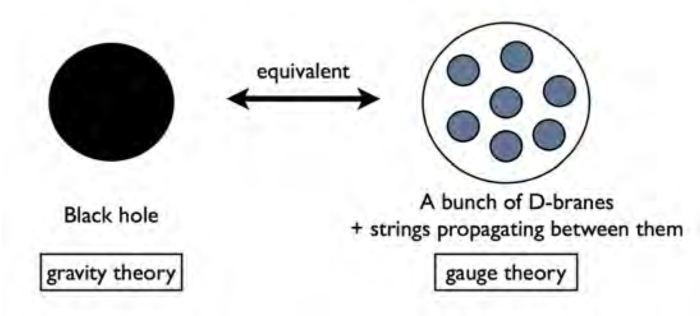

Figure 1: **The gauge/gravity duality conjecture.** Black holes in superstring theory are conjectured to be described by the dual gauge theory.

D-particles and the gauge/gravity duality

Superstring theory is a promising candidate for the theory of everything, which unifies the standard model of particle physics and gravity. In particular, it provides a consistent theory of quantum gravity, which is otherwise difficult to formulate due to non-renormalizable divergences at short distances. The theory contains two kinds of strings; closed strings and open strings. The former mediates gravitational force, while the latter mediates gauge interactions such as the electromagnetic force. Superstring theory also contains solitonic objects called D-branes [9], on which open strings can end. The dynamical property of D-branes including the oscillation of open strings is described by a gauge theory [13], which is a generalization of quantum electrodynamics.

Theoretical consistency requires that superstring theory should be defined in ten-dimensional space-time. In order to realize our four-dimensional space-time, one can choose the size of extra six dimensions to be very small. This procedure is called "compactification". In fact there are many ways to do it without spoiling the consistency, and by choosing the internal structure of the compactified extra dimensions appropriately, one can explain the variety of particles in four dimensions. However, since we are now interested in quantum effects of gravity, which become important at very short distances, we consider superstring theory without compactification. As a particular type of D-branes, we consider D-particles, which look like point-like objects in nine-dimensional space. It is known that a bunch of N D-particles is described by a gauge theory, in which all the fields are expressed as $N \times N$ matrices depending on time [13–15].

Superstring theory contains only one dimensionful parameter, which is conventionally written as $\alpha' = \ell^2$, where ℓ is the string length. In the low-energy limit, or equivalently in the $\alpha' \to 0$ limit, the oscillation of closed strings is dominated by the lowest energy states such as gravitons. If one further neglects quantum effects, the full superstring theory can be well approximated by supergravity, a generalized version of Einstein's gravity theory, which describes gravity in terms of the curvature associated with the space-time geometry. In supergravity, a bunch of N D-particles is expressed as an extremal black hole, which is stable and does not cause Hawking radiation. At finite temperature, the same system can be expressed as a non-extremal black hole. Since it has a positive specific heat, it cools down as it loses energy through Hawking radiation, and it eventually stabilizes into an extremal black hole at $T = 0$.

When N, the number of D-particles, is large, the size of the black hole is large and the geometry is weakly curved compared with the typical scale of quantum gravity. Hence quantum gravity effects can indeed be neglected. On the other hand, quantum gravity effects become important as N becomes small. In fact such effects can make the specific heat negative. In that case, the black hole heats up as it loses energy through Hawking radiation, and it will eventually evaporate completely. Thus this system at small N is relevant to the information loss paradox.

Unfortunately the full quantum nature of superstring theory has not yet been understood. However, according to the gauge/gravity duality conjecture, superstring theory in the presence of the black hole made of D-particles is equivalent to the gauge theory that describes the system of D-particles [16]. Since the gauge theory is well defined at arbitrary N, it captures the full quantum nature of superstring theory if the conjecture is correct. Furthermore, since the gauge theory is based on principles of quantum mechanics, it is clear that the information loss does not occur during the evaporation of the black hole. While there are many pieces of evidence for the gauge/gravity duality at $N = \infty$, where classical approximation is fully justified on the gravity side (See, for instance, ref [17]), very little is known about it at the level of quantum gravity.

Analysis on the gravity side

Let us start with an analysis on the gravity side. Readers who are not familiar with general relativity may jump directly to eq. (3), which represents the outcome of this analysis. The black hole, which is made of N D-particles in superstring theory, is described by a curved ten-dimensional space-time, which can be obtained as a solution to the classical equation of motion (or the "Einstein equation") for supergravity. The geometry is spherically symmetric in the nine-dimensional space, and the black hole is surrounded by an eight-dimensional surface called "event horizon". Once some object goes beyond the horizon from outside, it can never come out even with the speed of light. In particular, the metric near the horizon is given by [18, 19]

$$ds^2 = \alpha' \Big(- \frac{1}{\sqrt{H}} \, F dt^2 + \sqrt{H} \, \frac{1}{F} \, dU^2 + \sqrt{H} \, U^2 d\Omega_8^2 \Big) \, , \tag{1}$$

where U represents the radial coordinate and $d\Omega_8^2$ represents the line element of an eight-dimensional unit sphere. We have introduced the functions $H(U) = 240\pi^5\lambda/U^7$ and $F(U) = 1 - U_0^7/U^7$, where the two parameters λ and U_0 are related to the mass and charge of the black hole. Since $F(U)$ flips its sign at $U = U_0$, one finds that the horizon is located at $U = U_0$.

Now we consider quantum corrections to the classical geometry (1). Since superstring theory is defined perturbatively, one can calculate the leading quantum corrections to the geometry, which correspond to the $1/N^2$ corrections. It is well-known that the scattering amplitude involving four gravitons as asymptotic states gives nontrivial quantum corrections to the supergravity action at the leading order, which include quartic terms of the Riemann tensor [20]. By solving the equations of motion for supergravity including such corrections, one obtains the metric near the horizon as (Y. H., in preparation)

$$ds^2 = \alpha'\Big(- \frac{\sqrt{H_2}}{H_1} F_1\, dt^2 + \sqrt{H_2}\, \frac{1}{F_1}\, dU^2 + \sqrt{H_2}\, U^2 d\Omega_8^2 \Big) \ , \tag{2}$$

where $H_i = H + 5\pi^{11}\lambda^3 h_i/(24U_0^{13}N^2)$ for $i = 1, 2$ and $F_1 = F + \pi^6\lambda^2 f_1/(1152U_0^6 N^2)$. Here h_i and f_1 are functions of U/U_0, which can be determined uniquely. Note that the metric (2) reduces to (1) as $N \to \infty$, which corresponds to the limit of classical gravity. From this expression (2) for the metric, one finds that the position of the horizon is slightly shifted due to quantum effects. One also finds that a test particle feels a repulsive force near the horizon, which can be interpreted as the back-reaction of the Hawking radiation.

Given the geometry (2), one can evaluate the "energy" $\tilde{E}$ of the black hole as a function of temperature. (Strictly speaking, we evaluate the difference of the mass of the thermal non-extremal black hole from that of the extremal one. This quantity corresponds to the internal energy in the dual gauge theory, hence we use the word "energy".) For that we first calculate the entropy $\mathcal{S}$ of the black hole using Wald's formula, and obtain the "energy" $\tilde{E}$ by integrating the first law of thermodynamics, $d\tilde{E} = \tilde{T} d\mathcal{S}$. Here $\tilde{T}$ denotes the Hawking temperature, which can be derived from the geometry (2). Thus the "energy" of the black hole is evaluated as

$$\frac{1}{N^2} E_{\rm gravity} = 7.41\, T^{2.8} - 5.77\, T^{0.4} \frac{1}{N^2} \ , \tag{3}$$

up to $O(1/N^4)$ terms, where we have introduced dimensionless parameters $E_{\rm gravity} \equiv \lambda^{-1/3}\tilde{E}$ and $T \equiv \lambda^{-1/3}\tilde{T}$. In what follows, we call the energy normalized by $\lambda^{1/3}$ "effective energy". The first term in (3) can actually be obtained [21] at the classical level from the metric (1), and the second term represents quantum gravity corrections at the leading order. One finds that the specific heat $C = dE/dT$ becomes negative at sufficiently low T due to the second term. This means that the black hole becomes unstable due to the quantum gravity effects, and it actually evaporates.

In the above analysis, we have ignored the so-called α' corrections, which represent the effects due to the oscillation of strings. One can include these corrections to eq. (3) as has been done in ref [22] at $N = \infty$. Eq. (3) then becomes

$$\frac{1}{N^2} E^{\rm (full)}_{\rm gravity} = (7.41\, T^{2.8} + a\, T^{4.6} + \cdots) + (-5.77\, T^{0.4} + b\, T^{2.2} + \cdots)\frac{1}{N^2} + O\left(\frac{1}{N^4}\right) \ , \tag{4}$$

where a and b are unknown constants. The power of T for each term can be determined from dimensional analysis using some known results in superstring theory [23]. The gauge/gravity duality claims that eq. (4) should be reproduced by the dual gauge theory [16]. This has been tested at $N = \infty$, where the results from the gauge theory in the range $0.5 \le T \le 0.7$ can indeed be nicely fitted by the first two $O(N^0)$ terms in eq. (4) with $a = -5.58(1)$, thus confirming the gauge/gravity duality at the level of classical gravity [22] (See also refs [24–26] for related works.). The goal of our study is to see whether the gauge theory can reproduce the quantum gravity effects represented by the $1/N^2$ corrections in eq. (4).

Analysis on the gauge theory side

Let us turn to the analysis on the gauge theory side. The gauge theory that describes a bunch of N D-particles is defined by the action [14, 15]

$$S = \frac{N}{\lambda} \int_0^\beta dt \, \mathrm{tr} \left\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\psi_\alpha D_t \psi_\alpha - \frac{1}{2}\psi_\alpha (\gamma_i)_{\alpha\beta}[X_i, \psi_\beta] \right\} , \qquad (5)$$

where we have introduced the fields $X_i(t)$ ($i = 1, 2, \cdots, 9$) and $\psi_\alpha(t)$ ($\alpha = 1, 2, \cdots, 16$), which are $N \times N$ bosonic and fermionic Hermitian matrices depending on time t. Intuitively, the diagonal elements of X_i describe the positions of N D-particles in nine spacial directions, and the off-diagonal elements correspond to strings connecting different D-particles. The brackets $[\,\cdot\,,\,\cdot\,]$ represent the so-called commutator, which is defined by $[M_1, M_2] = M_1 M_2 - M_2 M_1$ for arbitrary matrices M_1 and M_2. We have also defined the covariant derivative $D_t = \partial_t - i\,[A_t,\,\cdot\,]$, where A_t is the gauge field represented by an $N \times N$ Hermitian matrix. The gamma matrices γ_i ($i = 1, \cdots, 9$) are 16×16 Hermitian matrices satisfying $\gamma_i \gamma_j + \gamma_j \gamma_i = 2\,\delta_{ij}$. As is usually done in studying thermal properties of gauge theories, the time coordinate t in eq. (5) actually represents "imaginary time", which is related to the real time $\tilde{t}$ through $\tilde{t} = -it$, and it is restricted to $0 \le t \le \beta \equiv 1/\tilde{T}$, where $\tilde{T}$ is the temperature, which should be identified with the Hawking temperature on the gravity side. The boundary conditions are taken to be periodic $A_t(t + \beta) = A_t(t)$, $X_i(t + \beta) = X_i(t)$ for bosonic matrices, and anti-periodic $\psi_\alpha(t + \beta) = -\psi_\alpha(t)$ for fermionic matrices. The partition function Z is defined as the sum of the Boltzmann factors $\exp(-S)$ for all field configurations, and the basic quantity we calculate is the internal energy, which is defined by $\tilde{E} = -(\partial/\partial\beta) \log Z$.

We put the system (5) on a computer as we have done in our previous works [22, 25]. We make a Fourier transform of each field with respect to time t, and introduce a cutoff Λ on the frequency. (Strictly speaking, we need to fix the gauge symmetry appropriately before we introduce a cutoff.) This method has practical advantage [27] over a more conventional method using lattice discretization [26], in which the matrices X_i and ψ_α are put on the sites of the lattice, whereas the gauge fields are put on the links connecting the sites. As far as the number of degrees of freedom is concerned, putting the frequency cutoff Λ corresponds to introducing a lattice with $(2\Lambda + 1)$ sites. In order to obtain a value in the continuum limit, we make an extrapolation to $\Lambda = \infty$. Although the fermionic matrices make the effective Boltzmann weight complex, we simply take the absolute value, which is shown to be a valid approximation in the present case [28].

In this work we focus on small values of N such as $N = 3, 4$ and 5 in order to probe the quantum gravity effects, which correspond to $1/N^2$ corrections. This causes a new technical difficulty, which was absent in previous works [22, 25] at large N such as $N = 17$. We observe that the eigenvalues of the bosonic matrices X_i start to diverge while we are sampling important field configurations that contribute to the partition function. This instability, however, can be interpreted as a physical one. It actually corresponds to the Hawking radiation of the black hole on the gravity side since the black hole is microscopically described by bound states of D-particles, and the positions of D-particles are represented by the eigenvalues of the bosonic matrices X_i in the gauge theory description. When N is sufficiently large, such bound states are stable [22, 25], which reflects the stability of the black hole in the absence of quantum gravity effects. When N becomes small, quantum gravity effects destabilize the black hole. Correspondingly, on the gauge theory side, we observe that the cluster of the eigenvalues becomes metastable as N becomes small.

In order to identify the metastable bound states, we first define a quantity

$$R^2 = \frac{1}{N\beta}\int_0^\beta dt \sum_{i=1}^{9} \mathrm{tr} X_i(t)^2 \ , \tag{6}$$

which represents the extent of the eigenvalue distribution of X_i. In Fig. 2, we show the histogram of R^2 for $N = 4$, $T = 0.10$, $\Lambda = 16$. A clear peak around $R^2 \sim 3.5$ confirms the existence of metastable bound states, while the non-vanishing tail at $4 \lesssim R^2 \lesssim 4.2$ reflects a run-away behavior associated with the instability.

This motivates us to calculate the effective internal energy by using only the configurations satisfying $R^2 < x$ for some x. We denote such a quantity $E(x)/N^2$ and plot it also in Fig. 2. We observe a clear "plateau" at the tail of the distribution of R^2. Therefore we use the height of this plateau as a sensible estimate of the effective internal energy of the metastable bound states.

In actual simulation we need to suppress the instability by adding a potential term $V_{\rm pot} = c\,|R^2 - R^2_{\rm cut}|$ for $R^2 > R^2_{\rm cut}$ to the action (5), where c should be sufficiently large to kill the instability. Note that the result for $E(x)/N^2$ presented in Fig. 2 does not depend on $R^2_{\rm cut}$ as far as $x < R^2_{\rm cut}$. We choose $R^2_{\rm cut}$ to be large enough to see the plateau behavior in $E(x)/N^2$. For instance, Fig. 2 is obtained with $c = 100$ and $R^2_{\rm cut} = 4.2$.

We repeat this analysis for all the parameter sets (N, T, Λ). We use $T = 0.08$, 0.09, 0.10, 0.11, 0.12 for $N = 3$, $T = 0.07$, 0.08, 0.09, 0.10, 0.11, 0.12 for $N = 4$ and $T = 0.08$, 0.09, 0.10, 0.11 for $N = 5$. Fitting the results E obtained for finite Λ using the ansatz $E = E_{\rm gauge} + \mathrm{const.}/\Lambda$, we obtain $E_{\rm gauge}$, which represents the effective internal energy in the continuum limit. The fitting was made with $\Lambda = 8, 10, 12, 14, 16$ for $T \geq 0.10$, $\Lambda = 10, 12, 14, 16$ for $T = 0.09, 0.08$ and $\Lambda = 12, 14, 16$ for $T = 0.07$.

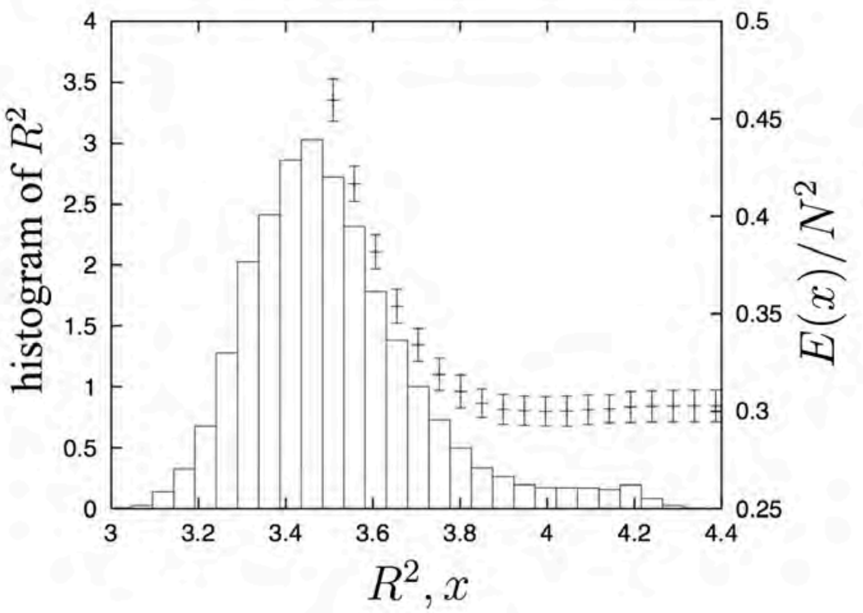

Figure 2: **The histogram of R^2 and the effective internal energy $E(x)/N^2$ obtained with configurations satisfying $R^2 < x$.** We show the results for $N = 4$, $T = 0.10$, $\Lambda = 10$ with the choice $R^2_{\rm cut} = 4.2$ and $c = 100$ for the cutoff potential. The peak of the histogram around $R^2 \sim 3.5$ represents the existence of the metastable bound states. The plateau behavior in $E(x)/N^2$ gives us a sensible estimate of the effective internal energy of the metastable bound states.

In Fig. 3 we plot our results for the effective internal energy in the continuum limit as a function of T for $N = 3, 4, 5$. (In the small box we show the extrapolation to $\Lambda = \infty$ for $N = 4$ and $T = 0.10$ as an example.) The curves represent the fits to the behaviors expected from the gravity side, which shall be explained later. We find that the internal energy increases as temperature decreases, which implies that the specific heat is negative. Such a behavior is possible since we are measuring the energy of the metastable bound states.

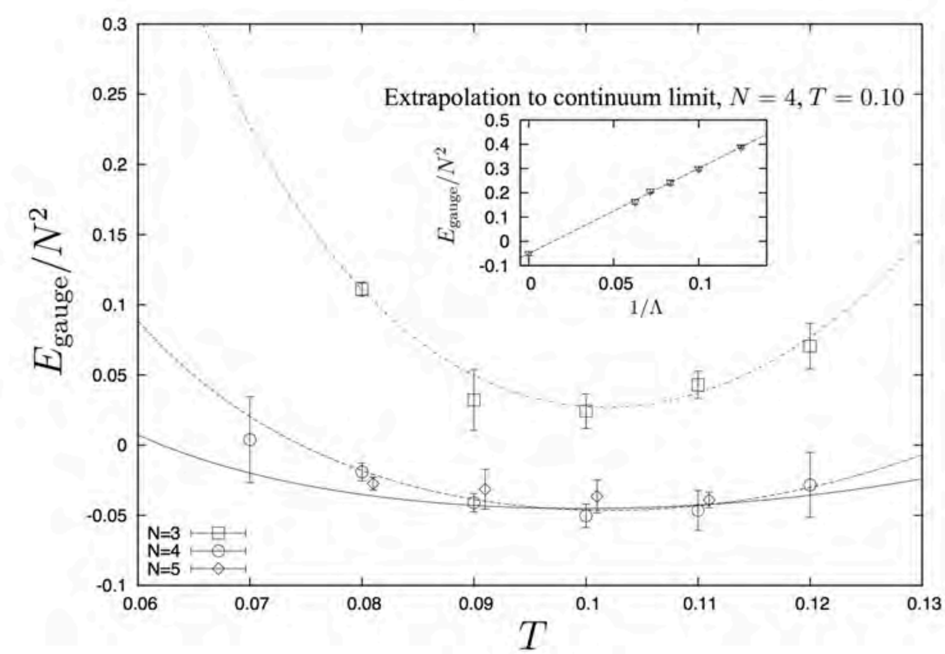

Figure 3: **The effective internal energy E_{gauge}/N^2 obtained for the metastable bound states in the continuum limit as a function of T.** Results for $N = 3$ (squares), $N = 4$ (circles) and $N = 5$ (diamonds) are shown. The curves represent the fits to the behaviors expected from the gravity side, which shall be explained later. The data points and the fitting curve for $N = 5$ are slightly shifted along the horizontal axis so that the data points and the error bars for $N = 4$ and $N = 5$ do not overlap. In the small box, we show an extrapolation to $\Lambda = \infty$ for $N = 4$ and $T = 0.10$.

Testing the gauge/gravity duality

Now we can test the gauge/gravity duality by comparing the results on the gauge theory side shown in Fig. 3 with the results on the gravity side represented by eq. (4). In the temperature regime $0.07 \leq T \leq 0.12$ investigated here, the terms with the coefficients a and b, which represent the α' corrections, can be neglected unless $|a| \gg 700$ and $|b| \gg 500$.

(As we mentioned earlier, a is obtained as $a = -5.58(1)$ by fitting the results from the gauge theory side [22] in the temperature regime $0.5 \le T \le 0.7$.) Therefore, we can actually test eq. (3) directly. In Fig. 4 we plot $(E_{\text{gauge}} - E_{\text{gravity}})/N^2$ against $1/N^4$ for $T = 0.08$ and $T = 0.11$. Our data are nicely fitted by straight lines passing through the origin. This implies that our result obtained on the gauge theory side is indeed consistent with the result (3) obtained on the gravity side including quantum gravity corrections. In the small box of the same figure, we plot E_{gauge}/N^2 against $1/N^2$. The curves represent the fits to the behavior $E_{\text{gauge}}/N^2 = 7.41\,T^{2.8} - 5.77\,T^{0.4}/N^2 + \text{const.}/N^4$ expected from the gravity side. We find that the $O(1/N^4)$ term is comparable to the $O(1/N^2)$ term. The fact that the $O(1/N^6)$ term is not visible from our data is therefore quite nontrivial and worth being understood from the gravity side. The agreement of similar accuracy is observed at other values of T.

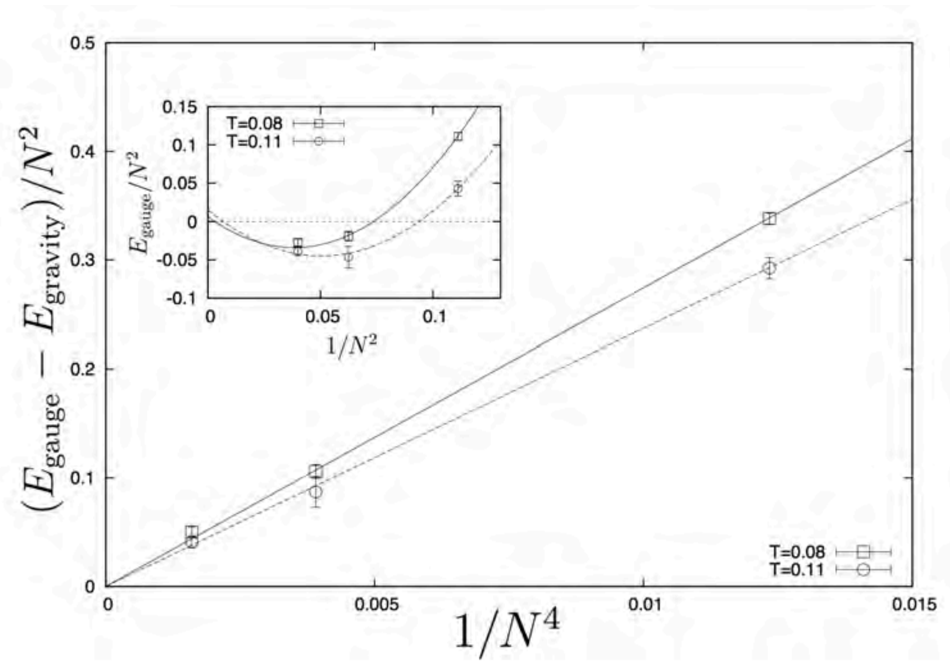

Figure 4: **The difference $(E_{\text{gauge}} - E_{\text{gravity}})/N^2$ as a function of $1/N^4$.** We show the results for $T = 0.08$ (squares) and $T = 0.11$ (circles). The data points can be nicely fitted by straight lines passing through the origin for each T. In the small box, we plot E_{gauge}/N^2 against $1/N^2$ for $T = 0.08$ and $T = 0.11$. The curves represent the fits to the behavior $E_{\text{gauge}}/N^2 = 7.41\,T^{2.8} - 5.77\,T^{0.4}/N^2 + \text{const.}/N^4$ expected from the gravity side.

As a further consistency check, we have also fitted our results for each T by $E_{\text{gauge}}/N^2 = 7.41\,T^{2.8} + c_1/N^2 + c_2/N^4$ leaving c_1 and c_2 as fitting parameters. In Fig. 5, we plot c_1 obtained by the two-parameter fit against T, which agrees well with $c_1 = -5.77\,T^{0.4}$.

As for the coefficient c_2 of the $O(1/N^4)$ terms, the prediction from the gravity side is given by $c_2 = c\,T^{-2.6} + \cdots$, where c is an unknown constant. In fact c_2 can be fitted, for

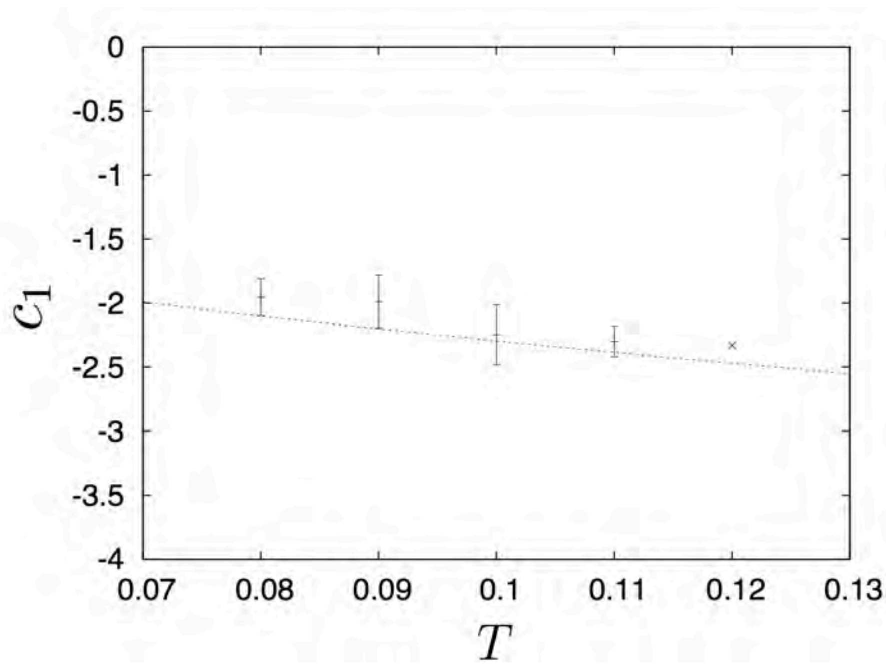

Figure 5: **The coefficient c_1 of the $O(1/N^2)$ term as a function of T.** Our results are consistent with the prediction $c_1 = -5.77\,T^{0.4}$ from the gravity side (the dotted line). The data point at $T = 0.12$ does not have an error bar since only two data points ($N = 3, 4$) were available for making a two-parameter fit.

instance, by $c_2 = cT^{-2.6} + \tilde{c}T^p$ with $c = 0.0340(12)$, $\tilde{c} = 0.17(23) \times 10^6$ and $p = 4.30(62)$. (The value for $\tilde{c}$ looks huge, but it is actually compensated by the high power of T within the temperature region investigated here.) Therefore we consider that the T dependence of c_2 is also consistent with the prediction from the gravity side. The curves in Fig. 3 represent $E_{\text{gauge}}/N^2 = E_{\text{gravity}}/N^2 + (cT^{-2.6} + \tilde{c}T^p)/N^4$ with the fitting parameters obtained above.

Summary and discussions

In this article we have given quantitative evidence for the gauge/gravity duality at the level of quantum gravity. In particular, we find that an evaporating black hole can be described by the dual gauge theory, which is based on fundamental principles of quantum mechanics. This provides us with an explicit example in which the information is not lost in an evaporating black hole.

Our work suggests a new approach to the quantum nature of gravity. Since the gauge/gravity duality is confirmed including quantum gravity effects, we can study various issues involving quantum gravity by using Monte Carlo simulation of the dual gauge theory. Thus the situation has become quite close to the studies of the strong interaction by simulating Quantum Chromodynamics on the lattice, which successfully explained the mass spectrum of hadrons [29] and the nuclear force [30] recently. We can now apply essentially the same method to study quantum gravity.

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References

[1] Hawking, S. W. Black hole explosions. *Nature* **248**, 30 (1974).

[2] Hawking, S. W. Particle creation by black holes. *Commun. Math. Phys.* **43**, 199 (1975). [Erratum-ibid. **46**, 206 (1976)].

[3] Bekenstein, J. D. Black holes and entropy. *Phys. Rev. D* **7**, 2333 (1973).

[4] Horowitz, G. T & Maldacena, J. M. The Black hole final state. *JHEP* **0402**, 008 (2004).

[5] Almheiri, A., Marolf, D., Polchinski, J. & Sully, J. Black holes: Complementarity or firewalls? *JHEP* **1302**, 062 (2013)

[6] Hawking, S. W. Breakdown of predictability in gravitational collapse. *Phys. Rev. D* **14**, 2460 (1976).

[7] Hawking, S. W. The unpredictability of quantum gravity. *Commun. Math. Phys.* **87**, 395 (1982).

[8] Strominger, A. & Vafa, C. Microscopic origin of the Bekenstein-Hawking entropy. *Phys. Lett. B* **379**, 99 (1996).

[9] Polchinski, J. Dirichlet branes and Ramond-Ramond charges. *Phys. Rev. Lett.* **75**, 4724 (1995).

[10] Maldacena, J. M. The Large N limit of superconformal field theories and supergravity. *Adv. Theor. Math. Phys.* **2**, 231 (1998).

[11] 't Hooft, G. Dimensional reduction in quantum gravity. `gr-qc/9310026`.

[12] Susskind, L. The world as a hologram. *J. Math. Phys.* **36**, 6377 (1995).

[13] Witten, E. Bound states of strings and p-branes. *Nucl. Phys. B* **460**, 335 (1996).

[14] Banks, T., Fischler, W., Shenker, S. H. & L. Susskind. M theory as a matrix model: A conjecture. *Phys. Rev. D* **55**, 5112 (1997).

[15] de Wit, B., Hoppe, J. & and Nicolai, H. On quantum mechanics of supermembranes. *Nucl. Phys. B* **305**, 545 (1988).

[16] Itzhaki, N., Maldacena, J. M., Sonnenschein, J. & Yankielowicz, S. Supergravity and the large N limit of theories with sixteen supercharges. *Phys. Rev. D* **58**, 046004 (1998).

[17] Minahan, J. A. *et al.* Review of AdS/CFT integrability. *Lett. Math. Phys.* **99**, 33 (2012).

[18] Horowitz, G. T. & and Strominger, A. Black strings and p-branes. *Nucl. Phys. B* **360**, 197 (1991).

[19] Gibbons, G. W. & Maeda, K. -i. Black holes and membranes in higher dimensional theories with dilaton fields. *Nucl. Phys. B* **298**, 741 (1988).

[20] Gross, D. J. & Witten, E. Superstring modifications of Einstein's equations. *Nucl. Phys. B* **277**, 1 (1986).

[21] Klebanov, I. R. & Tseytlin, A. A. Entropy of near extremal black p-branes. *Nucl. Phys. B* **475**, 164 (1996).

[22] Hanada, M., Hyakutake, Y., Nishimura, J. & Takeuchi, S. Higher derivative corrections to black hole thermodynamics from supersymmetric matrix quantum mechanics. *Phys. Rev. Lett.* **102**, 191602 (2009).

[23] Green, M. B., Russo, J. G. & Vanhove, P. Non-renormalisation conditions in type II string theory and maximal supergravity. *JHEP* **0702**, 099 (2007).

[24] Kabat, D. N. Lifschytz, G. & Lowe, D. A. Black hole thermodynamics from calculations in strongly coupled gauge theory. *Phys. Rev. Lett.* **86**, 1426 (2001).

[25] Anagnostopoulos, K. N., Hanada, M., Nishimura, J. & Takeuchi, S. Monte Carlo studies of supersymmetric matrix quantum mechanics with sixteen supercharges at finite temperature. *Phys. Rev. Lett.* **100**, 021601 (2008).

[26] Catterall, S. & Wiseman, T. Black hole thermodynamics from simulations of lattice Yang-Mills theory. *Phys. Rev. D* **78**, 041502 (2008).

[27] Hanada, M., Nishimura, J. & Takeuchi, S. Non-lattice simulation for supersymmetric gauge theories in one dimension. *Phys. Rev. Lett.* **99**, 161602 (2007).

[28] Hanada, M., Nishimura, J., Sekino Y. & Yoneya, T. Direct test of the gauge-gravity correspondence for Matrix theory correlation functions. *JHEP* **1112**, 020 (2011).

[29] Durr, S. *et al.* Ab-Initio determination of light hadron masses. *Science* **322**, 1224 (2008).

[30] Ishii, N., Aoki, S. & Hatsuda, T. The nuclear force from lattice QCD. *Phys. Rev. Lett.* **99**, 022001 (2007).