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1 **Inferring the Shallow Layered Structure at the**
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13 **Key Points:**

- 14 • We suggest a novel hyperbola-fitting technique that assumes an arbitrary permit-
 15 tivity distribution with respect to depth
 16 • The proposed method is used to map the lunar regolith at the Chang'E-4 land-
 17 ing site
 18 • A layered structure is revealed at the first 10 meters. A new stratigraphic model
 19 is suggested for the Von Kármán crater

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20 **Abstract**

21 The current paper investigates the shallow layers of the lunar regolith at the Chang'E-
 22 4 landing site. Four layers between 0-10 meters were identified using lunar penetrating
 23 radar. Based on these outputs, a revised stratigraphic model is suggested for the post-
 24 Imbrian ejecta at the Von Kármán crater. The layers were previously unseen due to the
 25 smooth boundaries between them. The revised model was inferred using an advanced
 26 hyperbola-fitting scheme. Applying conventional hyperbola-fitting to non-homogeneous
 27 media results in errors and inaccuracies that are often wrongly assumed to be negligi-
 28 ble. We propose a novel hyperbola-fitting scheme that is not constrained to homogeneous
 29 media and can be applied subject to any arbitrary one-dimensional permittivity distri-
 30 bution. Via this approach, we can estimate the permittivity profile of an investigated
 31 area and detect layered structures that were previously transparent to electromagnetic
 32 waves due to the gradational dielectric properties at their interfaces.

33 **Plain Language Summary**

34 The landing site of Cheng'E-4 is at the Von Kármán (VK) crater at the South Pole-
 35 Aitken (SPA) basin. SPA is the oldest and biggest basin on the Moon created at the early
 36 stages of its evolution by an impact that is believed that has penetrated the lunar crust
 37 and uplifted materials from the top mantle. Understanding the geology and stratigra-
 38 phy of SPA can help us understand cratering processes and shed a light on the evolu-
 39 tion of the Moon. In the current paper, we have used lunar penetrating radar data from
 40 the Chang'E-4 mission combined with a novel interpretation tool to reveal a previously
 41 unseen layered structure for the first ~ 10 m of the VK crater.

42 **Keywords**

43 South-Pole Aitken (SPA), Chang'E-4, Lunar Penetrating Radar (LPR), Ground
 44 Penetrating Radar (GPR), hyperbola-fitting.

45 **1 Introduction**

46 Ground penetrating radar (GPR) is a mature geophysical technique (Daniels, 2004)
 47 with a unique span of applications ranging from landmine detection (Feng et al., 2012;
 48 Giannakis et al., 2016) and concrete inspection (Wai-Lok Lai et al., 2018; Giannakis et
 49 al., 2020), to glaciology (Williams et al., 2014) and archaeology (Conyers, 2004). In plan-
 50 etary sciences, GPR has been applied both for satellite (Lauro et al., 2020) and in-situ
 51 measurements (Li et al., 2020), with promising results for mapping sub-glacial water bod-
 52 ies in Mars (Lauro et al., 2020), and for inferring the layered structure of the lunar re-
 53 golith (Lai et al., 2020; Li et al., 2020; Zhang et al., 2020).

54 Subject to the application and the employed measurement configuration, various
 55 GPR processing and interpretation techniques have been suggested over the years (Daniels,
 56 2004). From typical signal processing (Li et al., 2015; Cassidy, 2009) and linear Born ap-
 57 proximations (Boero et al., 2018), to machine learning (Giannakis et al., 2019) and full-
 58 waveform inversion (Meles et al., 2010). Within that context, hyperbola-fitting is con-
 59 sidered one of the most mainstream techniques for the interpretation of common-offset
 60 GPR data (Mertens et al., 2016). The simplicity and computational efficiency of hyperbola-
 61 fitting make it an appealing choice for mapping the dielectric properties of an investi-
 62 gated medium, and for estimating the coordinates of subsurface targets (Mertens et al.,
 63 2016).

64 Hyperbola-fitting has been used in both Chang'E-3 and Chang'E-4 missions (Li
 65 et al., 2020; Fa, 2020; Dong, Fang, et al., 2020; Dong, Feng, et al., 2020; Lai et al., 2019)
 66 for estimating the electric permittivity of lunar regolith and subsequently inferring its

67 density and mineralogical composition (Dong, Feng, et al., 2020; Li et al., 2020). Nonethe-
68 less, the underlying assumptions of hyperbola-fitting constrain its applicability, especially
69 in complex environments where permittivity varies with depth. To mitigate that, con-
70 ventional hyperbola-fitting is often complemented with Dix conversion (Dix, 1955; Dong,
71 Fang, et al., 2020) in order to transform the estimated bulk velocity to actual velocity.
72 Through a series of numerical examples, it is illustrated that this approach (Dong, Fang,
73 et al., 2020) has limited applicability for the lunar regolith and should be used with cau-
74 tion. To that extent, we present a novel hyperbola-fitting that tackles this problem by
75 simultaneously fitting multiple hyperbolas subject to any arbitrary 1D permittivity dis-
76 tribution.

77 The proposed scheme is applied to the Lunar Penetrating Radar (LPR) data col-
78 lected by the Yutu-2 rover during the first two lunar days of the Chang'E-4 mission at
79 the Von Kármán (VK) crater (Li et al., 2020). Four distinct layers –that were previously
80 not visible due to the smooth boundaries between them– were identified within the first
81 10 m. This outcome differs significantly from previous theories suggesting that the first
82 12 m of the landing site are fairly homogeneous, part of the weathered fine-grained re-
83 golith that lies on top of the ejecta from the Finsen crater (Zhang et al., 2020). Based
84 on the revised permittivity profile and the available literature on the geology of the Chang'E-
85 4 landing site, we suggest a new post-Imbrian stratigraphic model for the VK crater, in
86 which an approximately ~ 3 m weathered fine-grained layer is followed by $\sim 8 - 10$
87 meters of ejecta from the VK L and L' craters overlaying the ejecta from the Finsen crater.

88 2 The Chang'E-4 Landing Site

89 The Chinese lunar probe Chang'E-4, carrying the Yutu-2 rover, was the first human-
90 made object that landed on the far-side of the Moon on 3rd of January 2019 (Li et al.,
91 2019; Tang et al., 2020). The landing site is located at the South Pole-Aitkens (SPA)
92 basin – the oldest and biggest crater on the Moon (Huang et al., 2018; Hu et al., 2019;
93 James et al., 2019). The SPA basin is pre-Nectarian in age and has an elliptical shape
94 with an approximate diameter of 2100-2500 km (Moriarty et al., 2013). The transient
95 cavity of the SPA basin has been estimated between 840-1400 km (Potter et al., 2012;
96 Moriarty et al., 2013). The maximum excavation depth of lunar craters is approximately
97 10 % of their diameter (Stopar et al., 2017), which implies that the SPA basin excavated
98 up to 140 km through the lunar crust and into the mantle (Moriarty et al., 2013). This
99 premise is based on the maximum width of lunar crust ~ 60 km, as estimated by the
100 Gravity Recovery and Interior Laboratory (GRAIL) mission (Wieczorek et al., 2013),
101 which is in good agreement with seismic data from the Apollo missions (Khan, 2002).
102 The shallow mantle layer was most-likely melted during the impact (Moriarty et al., 2013)
103 and parts of it are expected to occur within the SPA basin, forming an underlying sheet
104 of non-crustal materials (Potter et al., 2012; Moriarty et al., 2013). These materials are
105 of paramount importance since they can constrain the composition of the upper man-
106 tle and provide an insight into the early evolution of the Moon (Moriarty et al., 2013).

107 Based on previous models of lunar evolution –that suggest an upper mantle pre-
108 dominantly composed of olivine (Yamamoto et al., 2010)– strong spectral signatures of
109 olivine were expected to be present within the SPA crater (Ivanov et al., 2018). Nonethe-
110 less, data from CLEMENTINE and SELENE did not support this premise (Tompkins
111 & Pieters, 1999; Matsunaga et al., 2008; Yamamoto et al., 2010), apart from small oc-
112 currences of olivine clusters (Yamamoto et al., 2010) most likely originated from crustal
113 materials, due to their location (the exterior of the SPA) and the high content of feldspar
114 in their near proximity (Moriarty & Pieters, 2018). The SPA is dominated by mafic ma-
115 terials and in particular with Mg-rich and low-Ca pyroxene (Moriarty & Pieters, 2018).
116 CLEMENTINE measurements reveal an inner zone with Fe abundance and an outer zone
117 with lower Fe content (Jolliff et al., 2000). Furthermore, using data from the Moon Minerol-
118 ogy Mapper (M^3), Moriarty & Pieters 2018 have divided the SPA into four zones. The

119 first zone is the inner SPA area called SPACA, with characteristic Ca-pyroxene abun-
 120 dant that lies at the center of the SPA. The second zone surrounds SPACA, and it is
 121 an area with Mg-rich pyroxenes. Based on spectral analysis of the central peaks of the
 122 craters within SPACA, strong indications were given to support the premise that SPACA
 123 lays on top of the Mg-rich area (Moriarty & Pieters, 2018). The third zone is a hetero-
 124 geneous annulus that consists of pyroxene and feldspar, and acts as the intermediate stage
 125 between the SPA and its exterior. The latter is the fourth zone, a mafic-free area with
 126 high content of feldspar, similar to lunar highlands (Moriarty & Pieters, 2018).

127 The landing site of Chang'E-4 is within the Mg-rich annulus and in particular in the
 128 interior of the VK crater (177.588°E, 45.4578°S). VK is an elliptical crater (Zhang et al.,
 129 2020) with approximately ~ 186 km diameter (Huang et al., 2018). The age of VK was
 130 estimated pre-Nektarian (Huang et al., 2018) and recent studies have placed it at ~ 4.2 Ga
 131 (Lu et al., 2021), very close to the formation of SPA (Lu et al., 2021). The creation of
 132 Leibnitz crater affected the north part of VK and contributed to the ejecta layer prior
 133 to the Imbrian basaltic flood (Huang et al., 2018). Ejecta from Alder crater (dated at
 134 ~ 3.5 Ga (Lu et al., 2021)) are also expected to the pre-basaltic layers (Huang et al., 2018).
 135 The VK crater was flooded with basalts during the Imbrian period (Paskert et al., 2018)
 136 around ~ 3.2 – 3.3 Ga. Subsequently, ejecta from the Finsen crater were deposited at the
 137 end of Imbrian and early Eratosthenian (~ 3.1 Ga (Lu et al., 2021)). Recent studies sug-
 138 gest that Orientale crater might have added to the post-Imbrian VK layers as well (Xiao
 139 et al., 2021). Subsequently, the Eratosthenian craters VK L and L' were formed (Zhang
 140 et al., 2020). The VK, Leibnitz, Alder, VK L, and L' lay within the Mg-pyroxene an-
 141 nulus while Finsen is within SPACA (Moriarty & Pieters, 2018).

142 Geological context suggests that the craters VK L, L', Finsen and Orientale have
 143 contributed to most of the post-Imbrian ejecta layers of the VK crater (Huang et al., 2018;
 144 Di et al., 2019; Xiao et al., 2021). The size of the ejecta from the Finsen crater is esti-
 145 mated –via numerical simulations (Di et al., 2019)– at ~ 30 meters. This is not in good
 146 agreement with the results obtained using dark-halo and non-dark halo craters (Li et al.,
 147 2020) that suggest a thicker post-basaltic layer, probably due to the presence of Ori-
 148 entale ejecta (Xiao et al., 2021). Nonetheless, contradicting data (Yue et al., 2020) place
 149 the date of Orientale to be older than the Imbrian basaltic flood, which implies that there
 150 might be another source that contributed to the post-basaltic VK layers.

151 The surface of the landing site is smooth with a small amount of boulders, most
 152 of them being glassy fragments and breccias from secondary craters (Lin et al., 2020).
 153 From in situ reflectance data, the visible surface at the landing site is not olivine-pyroxene
 154 rich and consists of 56-72% plagioclase, similar to lunar highlands (Hu et al., 2019; Li
 155 et al., 2019) with Mg-rich orthopyroxene (Gou et al., 2020). The thickness of the regolith
 156 (weathered top soil) is estimated using LROC NAC images at ~ 2.5 – 7.5 m (Huang
 157 et al., 2018). Based on the M^3 reflectance data, it is estimated that below the top weath-
 158 ered soil, lays a low-calcium pyroxene (LCP) layer ranging from ~ 8 – 13 m followed
 159 by a high-calcium pyroxene (HCP) layer from ~ 13 – 53 m (Huang et al., 2018). Be-
 160 low that, the Imbrian basalt deposits are expected to overlay the ejecta from the Alder
 161 and Leibnitz craters on top of the brecciated bedrock from the VK impact (Huang et
 162 al., 2018).

163 Further insights on the ejecta at the VK crater are provided by the LPR mounted
 164 to the Yutu-2 rover of the Chang'E-4 mission (Li et al., 2020). The first attempt to ex-
 165 amine the lunar surface with in-situ LPR equipment occurred during the Chang'E-3 mis-
 166 sion on the near side of the moon (Lai et al., 2019). Similar antenna configurations were
 167 employed for both Chang'E-3 and Chang'E-4 missions (Li et al., 2020). In particular,
 168 two antennas with 500 MHz central frequency (at the bottom of the rover), and one low
 169 frequency antenna (mounted at the back of the rover) with 60 MHz central frequency
 170 (Li et al., 2020). The low frequency antenna in the Chang'E-4 mission gave thin indi-
 171 cations of four different lava flows that probably occurred during the Imbrian period (Lai

172 et al., 2020). Unfortunately, the low frequency data in both missions suffer from ring-
 173 ing noise due to the coupling between the antenna and the rover, which resulted in er-
 174 roneous reflections and noisy data (Li et al., 2018; Zhang et al., 2020). In contrast to the
 175 Chang'E-3 landing site (Lai et al., 2019), in the VK crater, the ilmenite content is fairly
 176 low, making the ejecta layers transparent to LPR (Dong, Fang, et al., 2020). This re-
 177 sulted in good quality data that clearly demonstrated a complex layered structure for
 178 the first 50 meters of the VK crater (Zhang et al., 2020; Li et al., 2020). In addition, us-
 179 ing a conventional hyperbola-fitting (assuming a homogeneous medium) with Dix con-
 180 version, the electric permittivity of the ejecta layers was estimated, and furthermore used
 181 to infer the mineralogical (Fe and Ti content) (Li et al., 2020) and the mechanical (den-
 182 sity) properties of the lunar regolith (Dong, Fang, et al., 2020; Dong, Feng, et al., 2020),
 183 based on semi-empirical formulas fine-tuned for lunar soils (Olhoeft & Strangway, 1975;
 184 Carrier et al., 1991; Hickson et al., 2018). The relative electric permittivity at the land-
 185 ing site monotonically increases from $\sim 3 - 6$ with respect to depth, as estimated us-
 186 ing typical hyperbola fitting (Dong, Fang, et al., 2020). This corresponds to a density
 187 that starts from $\sim 1 \text{ gr/cm}^3$ at the surface and reaches 2.5 gr/cm^3 at 50 m depth (Dong,
 188 Fang, et al., 2020).

189 3 Methodology

190 3.1 Advanced hyperbola-fitting

In this section, a novel hyperbola-fitting framework is described, capable of deal-
 ing with half-spaces with arbitrary 1D permittivity distributions $\epsilon(y)$ (see Figure 1). Sim-
 ilar to typical hyperbola-fitting, in order to avoid non-uniqueness (Mertens et al., 2016;
 Giannakis et al., 2019), the proposed scheme assumes that the radius (R) of the inves-
 tigated target equals with zero. Subject to a varying velocity with depth, the two way
 travel time t that it takes for the wave to travel from the point $\vec{B} = \langle x, y \rangle$ to the point
 $\vec{A} = \langle x_0, d \rangle$ via the parametric curve $\vec{q}(m) = \langle q_x(m), q_z(m) \rangle$ (where $m \in [0 - 1]$ and
 d is the depth of the target) can be calculated using a scalar line integral over $\vec{q}(m)$

$$t = \frac{2}{c_0} \int_0^1 \sqrt{\epsilon(y)} \left\| \frac{\partial \vec{q}(m)}{\partial m} \right\| dm. \quad (1)$$

Given a specific velocity structure, the path $\vec{q}(m)$ can be calculated using Fermat's prin-
 ciple (Aldo, 1996). The notation $\left\| \frac{\partial \vec{q}(m)}{\partial m} \right\|$ is used to denote the norm of the first deriva-
 tive of the parametric curve $\vec{q}(m)$ with respect to the parameter $m \in [0 - 1]$. It is shown
 that if we simplify equation (1) and make the assumption that the path $\vec{q}(m)$ is the straight
 line that connects the antenna to the center of the target, it leads to an elegant and com-
 putationally efficient formulation without compromising accuracy (more details are given
 in 3.2). The straight line that connects the antenna to the target can be expressed via
 the parametric curve $\vec{q}(m) = \vec{A} + (\vec{B} - \vec{A})m$. Substituting this into equation (1)
 results in

$$t = \frac{2 \|\vec{B} - \vec{A}\|}{c_0} \int_0^1 \sqrt{\epsilon(y)} dm. \quad (2)$$

The linear path of the integral in equation (2) can be written as $\vec{q}(m) = \langle x_i + m(x_0 - x_i), y_i +$
 $m(d - y_i) \rangle$, where x_i, y_i are the coordinates of the antenna at the i th position. Conse-
 quently, the y variable in equation (2) can be substituted by $y = y_i + m(d - y_i)$, which
 implies that $\partial m = \frac{\partial y}{d}$ and that $y = d$ for $m = 1$. Therefore, equation (2) can be re-
 written as

$$t = \frac{2 \|\vec{B} - \vec{A}\|}{c_0 d} \int_0^d \sqrt{\epsilon(y)} dy. \quad (3)$$

Solving the integral numerically yields

$$t \approx \frac{2 \|\vec{B} - \vec{A}\|}{c_0 d} \sum_{s=0}^Q \sqrt{\epsilon(s \cdot \Delta y)} \Delta y \quad (4)$$

where Δy is the discretization step and $Q = d/\Delta y$. Notice that the summation term $N(d, \epsilon) = \sum_{s=0}^Q \sqrt{\epsilon(s \cdot \Delta y)} \Delta y$ is independent of the position of the antenna and needs to be calculated just once. The final formulation for the proposed scheme is given by

$$t \approx \frac{2\|\vec{B} - \vec{A}\|}{c_0 d} N(d, \epsilon), \tag{5}$$

where the only unknowns are the permittivity function $\epsilon(y)$ and the depth of the target d . The parameter x_0 can be easily derived from the apex of the hyperbola at the measured B-Scan. Subject to a given $\epsilon(y)$, the depth of the target is calculated using the apex of the hyperbola (x_0, t_0) , where $\|\vec{B} - \vec{A}\| = d$

$$t_0 \approx \frac{2}{c_0} N(d, \epsilon). \tag{6}$$

191 For a given $\epsilon(y)$, the only unknown in equation (6) is the depth d that is estimated numerically using the bisection method. Notice that both equation (6) and the summation
 192 $N(d, \epsilon)$ in equation (4) need to be evaluated just once. The only term in equation (5)
 193 that needs to be updated as the scan progresses is the distance $\|\vec{B} - \vec{A}\|$. To summarize,
 194 given a permittivity distribution $\epsilon(y)$ and the apex of a hyperbola $[x_0, t_0]$, the depth
 195 d of the target is estimated by numerically solving equation (6) using the bisection method.
 196 Subsequently, $N(d, \epsilon)$ is evaluated and furthermore used in equation (5) to calculate the
 197 arrival times $\mathbf{t} \in \mathbb{R}^n$.
 198

199 The proposed scheme utilizes numerous hyperbolas and tries to find the optimum
 200 $\epsilon(y)$ that simultaneously minimizes $\min_{\epsilon(y)} \sum_{i=1}^Z \|\mathbf{t}_i - \mathbf{T}_i\|$, where $\mathbf{T}_i \in \mathbb{R}^{n_i}$ and $\mathbf{t}_i \in \mathbb{R}^{n_i}$
 201 represent the measured and predicted arrival times for the i th hyperbola, Z is the total
 202 number of the employed hyperbolas and n_i is the number of discretisation points for
 203 the i th hyperbola. To further simplify the problem, the permittivity is discretised with
 204 K equidistant points and subsequently a cubic interpolation is applied to map ϵ with respect
 205 to y in a continuous manner. Therefore, the minimization is re-written as $\min_{\epsilon^{(k)}} \sum_{i=1}^Z \|\mathbf{t}_i -$
 206 $\mathbf{T}_i\|$ with only K number of unknowns. This is a non-linear and non-convex problem that
 207 can be solved using global optimizers. A Particle Swarm Optimisation (PSO) (Kennedy
 208 & Eberhart, 1995), with 50 particles and uniform PSO parameters, was proven to be very
 209 efficient for reconstructing $\epsilon(y)$, given a sufficient number of measured hyperbolas. The
 210 number of equidistant points K is estimated by plotting the Error- K curve. This approach
 211 is based on the L-curve method (Hansen, 1992) that tries to balance between accuracy
 212 and constraints. Within that context, we choose the K value for which the solution balances
 213 accuracy and simplicity. In particular, the minimization is executed multiple times
 214 with increasing K until the error starts to converge. The K value is chosen at the earliest
 215 point of convergence. Greater K values can potentially result (if a sufficient number
 216 of hyperbolas is not present) in unnecessary complicated permittivity structures without
 217 increasing the fitting accuracy.

218 3.2 Numerical experiments

219 Two numerical 2D case studies (illustrated in Figure 2) are used in this section to
 220 evaluate the performance of the proposed scheme. Both models are non-dispersive, non-
 221 conductive, and non-magnetic, with a varying permittivity with respect to depth $\epsilon(y)$.
 222 Nine perfect electric conductors (PEC) are distributed randomly within a $2 \times 1 \text{ m}^2$ domain.
 223 The targets have a cylindrical shape with 5 cm diameter and their main axis is
 224 perpendicular to the acquisition line. Measurements are taken every 2 cm along the x-
 225 axis using a line source with 1 GHz central frequency. The offset between the transmitter
 226 and the receiver is 1 cm. The numerical simulations were executed using gprMax (Warren
 227 et al., 2016; Warren et al., 2019), an open source electromagnetic solver that uses a second
 228 order (in both space and time) finite-difference time domain (FDTD) method (Yee,

229 1966). The spatial discretization step of the FDTD grid is $\Delta x = \Delta y = 5$ mm, and
 230 the time step Δt is calculated using the Courant limit (Taflove & Hagness, 2000). The
 231 boundaries of the domain are truncated using the recursive integration perfectly matched
 232 layer (Giannopoulos, 2008).

233 From Figure 2, it is apparent that even in these clinical clutter-free numerical exper-
 234 iments, the reflections from the layers are very weak and not visible in the measured
 235 radargrams. This is due to the smooth transition between the layers that can greatly de-
 236 crease their reflection coefficient (Bano, 2006; Diamanti et al., 2014). This gives the false
 237 impression that a medium is homogeneous when in fact it can be as complex as Model
 238 1 (see Figure 2), with four clear and distinct layers. This is very important when inter-
 239 preting radargrams from the lunar regolith, where smooth transitions between layers are
 240 expected due to space weathering and the reworking of the materials during crater for-
 241 mation.

242 The proposed scheme and the typical hyperbola-fitting with Dix conversion (Dong,
 243 Fang, et al., 2020) were applied to the radargrams shown in Figure 2. In both models,
 244 the proposed methodology outperforms conventional hyperbola-fitting, and manages to
 245 sufficiently estimate the permittivity profile and the underlying layered structure in an
 246 efficient manner (see Figure 2). Small errors observed in Figure 2 can be due to: the linear-
 247 path simplification; manual picking of the hyperbolas (Ding et al., 2020); non-accurate
 248 time-zero correction (Yelf, 2004); and/or non-ideal targets i.e. $R \neq 0$.

249 4 Results

250 The proposed methodology is applied to the high frequency data collected by the
 251 Yutu-2 rover at the VK crater during the first two lunar days of the Chang’E-4 mission
 252 (Li et al., 2020). During the first two lunar days, the rover followed an irregular path
 253 and managed to cover ~ 106 m (Li et al., 2020). The current paper focuses on the first
 254 150 ns of the scan in order to effectively map the shallow layers ($\sim 10 - 12$ m) of the
 255 regolith. Based on the results, a revised stratigraphy for the VK crater is proposed that
 256 takes into account a previously unseen layered structure within the first ~ 10 m of lu-
 257 nar regolith.

258 4.1 Lunar penetrating radar results

259 The radargram was processed using a typical GPR processing pipeline that involves
 260 zero-time correction, dewow, time-gain (exponential gain), and background removal (Cassidy,
 261 2009). The resulting B-Scan for the first 150 ns is illustrated in Figure 3 (Li et al., 2020).
 262 The overall signal to clutter ratio is substantially higher compared to Chang’E-3 mis-
 263 sion (Lai et al., 2019; Li et al., 2020) (potentially due to lack of ilmenite) which results
 264 in clear hyperbolic features that can be utilized to deduce the shallow layered structure
 265 at the first 10-12 m of the landing site.

266 Figure 3_A illustrates the resulting permittivity profile using the proposed advanced
 267 hyperbola-fitting subject to the hyperbolas shown in Figure 3_B. It is evident that there
 268 is a layered structure with four layers in the first 10 m of the regolith. The first and the
 269 third layers have low permittivity values while the second and the fourth layers have per-
 270 mittivity up to $\epsilon \approx 10$ (see Figure 3_A). Typical lunar soils have low permittivity val-
 271 ues although there are reported high-density lunar samples with relative permittivity up
 272 to $\epsilon \approx 10$ (Chung et al., 1970; Olhoeft & Strangway, 1975).

273 We would like to highlight that current knowledge regarding the permittivity of
 274 lunar soils is based primarily on shallow samples brought back to Earth during the Apollo
 275 missions. Superficial lunar samples are not representative of deeper layers since they are
 276 exposed to space weathering which results in an increased porosity and vitrification (Nash

277 & Conel, 1973). Moreover, the semi-empirical models tuned for lunar soils are primar-
 278 ily based on those samples (Chung et al., 1970; Frisillo et al., 1975; Carrier et al., 1991;
 279 Shkuratov & Bondarenko, 2001), making them unreliable for estimating the dielectric
 280 properties of deeper ejecta. Estimation of the dielectric properties of deeper layers is still
 281 an ongoing research area that is primarily based on LPR measurements and typical hyperbola-
 282 fitting (Dong, Fang, et al., 2020). As shown in section 3, typical hyperbola-fitting is not
 283 a reliable approach when applied to inhomogeneous media, and therefore the estimated
 284 permittivities using conventional hyperbola-fitting should be used with caution.

285 **4.2 Stratigraphy modeling of the Chang'E-4 landing site**

286 The suggested stratigraphy model is based on the LPR results shown in Figure 3A
 287 and the following premises:

- 288 • The thickness of the weathered top soil is $\sim 2.5\text{--}7.5$ m (Huang et al., 2018) which
 289 is consistent with the average weathering rate (~ 1.5 m/Ga) derived from the Apollo
 290 missions (Gou et al., 2021).
- 291 • Finsen, VK L and L' craters are the predominant sources of the post-Imbrian ejecta
 292 in the VK crater (Huang et al., 2018; Zhang et al., 2020).
- 293 • Finsen ejecta at the landing site are estimated via numerical simulations at ~ 35
 294 m (Di et al., 2019).
- 295 • Finsen crater was developed before VK L and L' craters (Zhang et al., 2020).
- 296 • Finsen crater is within the SPACA region and therefore it is expected that its ex-
 297 cavated materials have an increased HCP/LCP ratio (Moriarty & Pieters, 2018).
 298 The peak of the Finsen crater has a low HPC/LCP ratio (Ling et al., 2019), nonethe-
 299 less, the peak of craters is created in a rebound process that uplifts lower mate-
 300 rials (Morgan et al., 2016) i.e. materials from the underlying Mg-rich anulus which
 301 has low HPC/LPC ratio (Moriarty & Pieters, 2018).
- 302 • The ejecta materials from VK L and L' craters have low HCP/LCP ratio (Ling
 303 et al., 2019).
- 304 • Below the weathered top soil there is an LCP layer down to ~ 13 m (Huang et
 305 al., 2018).
- 306 • Below the LCP layer there is a thick layer (> 30 m) with high HCP/LCP ratio
 307 (Huang et al., 2018).
- 308 • There is a clear sharp boundary observed on LPR data (Zhang et al., 2020) at \sim
 309 13 m, most-likely between the LCP and the HCP layer.

310 The proposed stratigraphy model suggests that the HCP layer overlaying the Im-
 311 brian basalts is the ejecta from the Finsen crater (Huang et al., 2018) (and maybe Ori-
 312 entale crater too(Xiao et al., 2021)). This premise is consistent both with the size of this
 313 layer (as predicted by numerical simulations (Di et al., 2019)) and with the chemical com-
 314 position of the Finsen crater (Moriarty & Pieters, 2018). On top of the Finsen ejecta,
 315 it is expected to encounter ejecta from Eratosthenian post-Finsen craters. A homoge-
 316 neous weathered layer with 12 m width as suggested by Zhang et al., 2020 is not con-
 317 sistent with LROC NAC images (Huang et al., 2018) and by the layered structure re-
 318 vealed by the proposed hyperbola-fitting scheme (see Figure 3A). Therefore, we suggest
 319 that the top $\sim 10 - 12$ m of the landing site consists of ejecta from the VK L and L'
 320 craters. This is in good agreement with the LCP content of the VK L and L' craters and
 321 with the layered structure illustrated in Figure 3A. In addition, a 12-13 m regolith in-
 322 dicates a weathering rate of $\sim 3\text{--}4$ m/Ga which is twice as fast compared to the ones
 323 derived from the Apollo missions (apart from the landing site of Apollo 16) (Gou et al.,
 324 2021). The evolution of the post-basaltic flood ejecta of VK crater is shown in Figure
 325 3C. The ejecta of VK L' (≈ 5.5 m) were deposited on top of the Finsen ejecta at early
 326 Eratosthenian. Space weathering degraded the first ~ 1.5 m of the ejecta decreasing its
 327 density and consequently its electric permittivity (due to the causal relationship between

328 permittivity and density (Chung et al., 1970; Olhoeft & Strangway, 1975)). The width
 329 of the VK L' regolith is in good agreement with the literature which suggests that rapid
 330 weathering is expected at young ejecta, a phenomenon that has also been observed at
 331 the Chang'E-3 landing site (Gou et al., 2021). The ejecta from VK L is subsequently de-
 332 posited on top of the weathered layer creating a top layer with ~ 6 m width. The long
 333 weathering process, from early Eratosthenian till now, gave rise to a ~ 3 m of loose lu-
 334 nar soil with low electric permittivity as predicted by Figure 3A. This is in good agree-
 335 ment with the LROC NAC images (Huang et al., 2018) and also with the average weath-
 336 ering rate (~ 1.5 m/Ga) derived from the Apollo missions (Gou et al., 2021).

337 5 Conclusions

338 A novel interpretation tool was described capable of estimating the permittivity
 339 profile of the shallow lunar surface using lunar penetrating radar. The validity and the
 340 superiority of the suggested scheme compared to typical hyperbola-fitting was demon-
 341 strated via a set of numerical experiments that clearly shown that the proposed scheme
 342 is capable of reconstructing complicated permittivity profiles using the shape of multi-
 343 ple hyperbolas as the only inputs. The proposed methodology is suitable for any arbi-
 344 trary one-dimensional permittivity distribution, which makes it an appealing choice for
 345 inferring the mechanical and mineralogical properties of lunar regolith. The advanced
 346 hyperbola-fitting was applied to the high frequency data collected during the first two
 347 lunar days of the Chang'E-4 mission. The resulting permittivity profile indicates a lay-
 348 ered structure within the first 10 meters of the regolith. These shallow layers are not vis-
 349 ible in the measured radargram due to the smooth boundaries between them, making
 350 them undetectable using traditional signal processing approaches. It is argued that the
 351 multiple layers detected within the shallow lunar regolith can be the ejecta of the Er-
 352 atosthenian craters Von Kármán L and L', laying on top of the late-Imbrian ejecta of
 353 Finsen crater.

354 Data Availability Statement

355 The Chang'E-4 Lunar Penetrating Radar data are available from the Data Pub-
 356 lishing and Information Service System of China Lunar Exploration Program [http://](http://moon.bao.ac.cn/searchOrder_dataSearchData.search)
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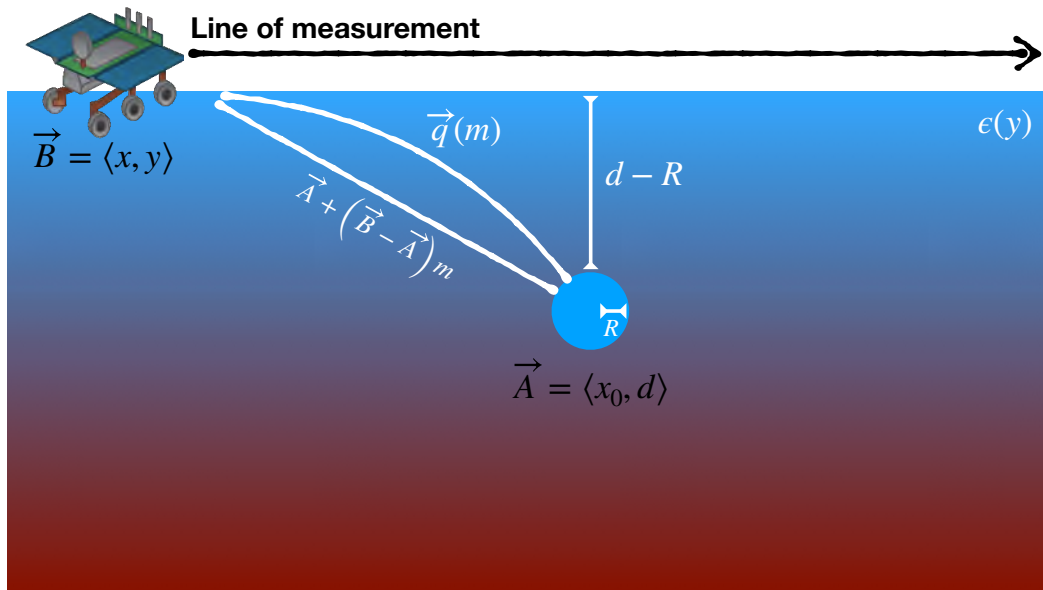


Figure 1. A simple scenario investigating a cylindrical target with radius R buried in a half-space subject to a 1D electric permittivity distribution with respect to depth $\epsilon(y)$. The vector positions of the center of the target and the antenna are $\vec{A} = \langle x_0, d \rangle$ and $\vec{B} = \langle x, y \rangle$ respectively. The distance between the antenna and the surface of the target equals $\|\vec{A} - \vec{B}\| - R$. For both lunar and Earth applications, the permittivity often increases with depth and therefore the velocity is expected to decrease. Due to that, the wave will follow a path similar to the parametric curve $q(m)$ with $m \in [0 - 1]$. The parametric equation of the line that connects the point of measurement to the centre of the target is given by $\vec{A} + (\vec{B} - \vec{A})m$.

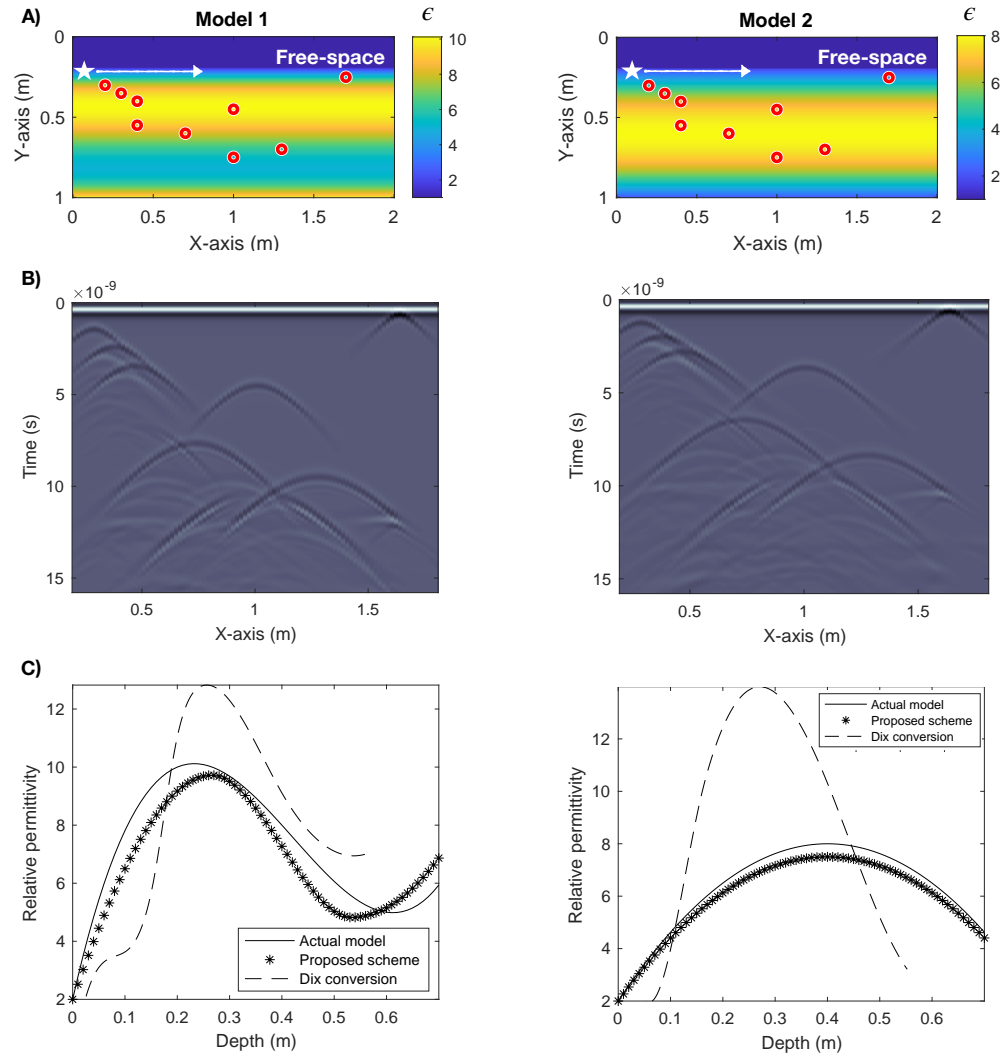


Figure 2. A) The investigated numerical experiments. Nine cylindrical targets are buried in two media with varying permittivity with respect to depth. Measurements are taken every 2 cm (from left to right) using a ground-coupled line source (white star) with 1 GHz central frequency. B) Resulting B-Scans. It is evident that due to the smooth boundaries between the layers, no reflections are visible on the resulting radargrams. The shapes of the hyperbolas are the only features that can be used to infer the permittivity profile. C) The resulting permittivity profile for the models shown in Figure 2A.

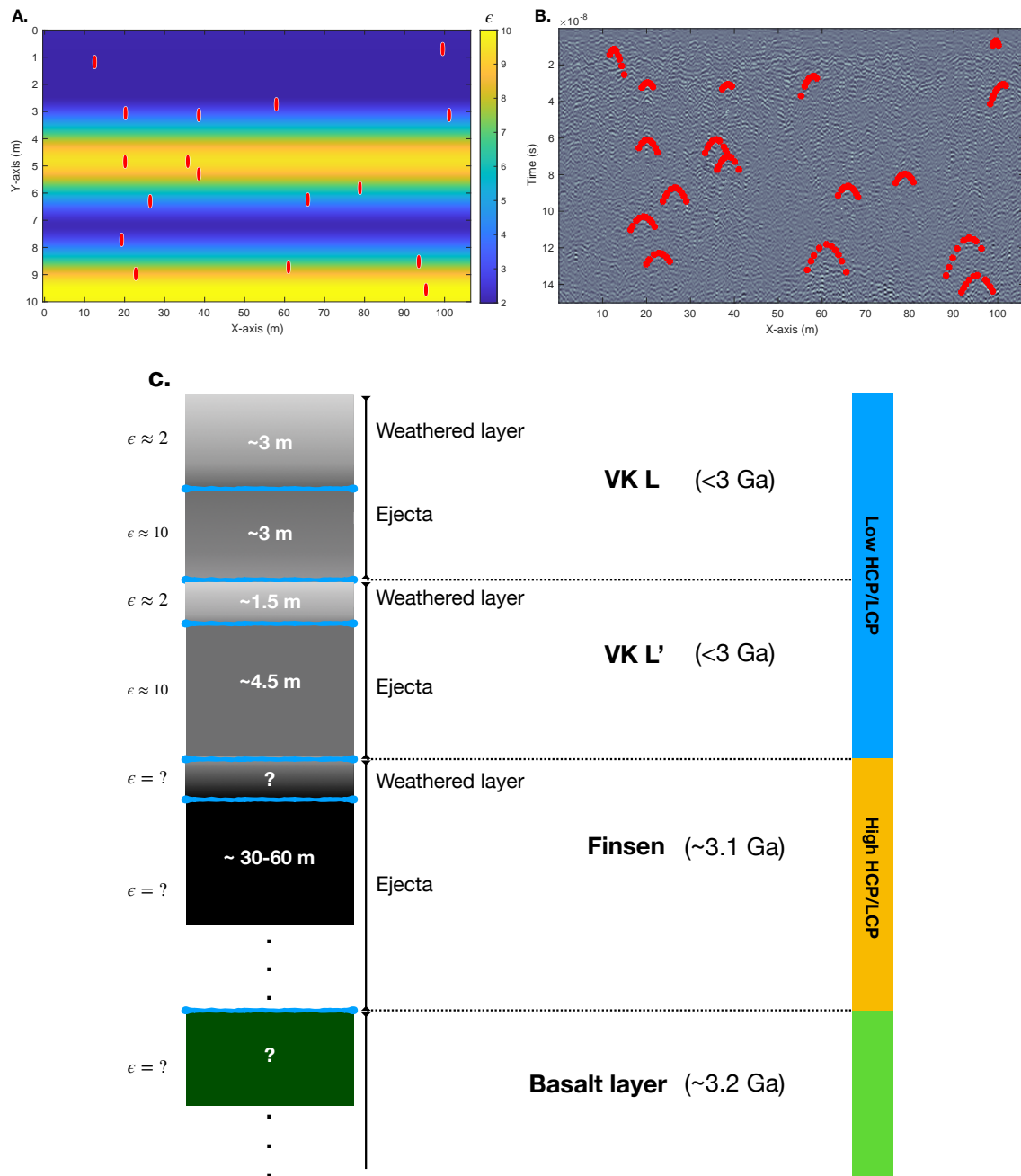


Figure 3. A) The resulting permittivity profile $\epsilon(y)$ at the landing site of Chang'E-4 mission using the advanced hyperbola fitting. The coordinates of the investigated targets are illustrated with red dots. B) The fitted hyperbolas subject to the permittivity profile shown in Figure 3A. C) The proposed stratigraphy model for the Chang'E-4 landing site. The first ~ 6 m consists of a top weathered layer overlaying the ejecta from VK L crater. Below that, is a low permittivity layer that corresponds to the weathered ejecta of the VK L' crater. The VK L' ejecta extends to ~ 12 m depth, where the Finsen and Alder ejecta lay on top of the Imbrian basaltic layer. Dates are based on (Lu et al., 2021) and the chemical composition on (Huang et al., 2018).